Grade 9

CONTENT BOOKLET: TARGETED SUPPORT MATHEMATICS

Term 2

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

TOPIC 1: CONSTRUCTION OF GEOMETRIC FIGURES

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Space and Shape' which counts for 30% in the final exam.
- The unit covers geometric constructions. The majority of the work was covered in Grade 8 so a large amount of the time allocated can be used to revise and consolidate what was done then.
- The purpose of teaching constructions is to help learners visualise geometry. When learners construct the shapes themselves they gain a better understanding of each shape which leads to a better understanding of the theorems.
- Ensure that all the necessary equipment is available beforehand either for learners to work with individually or in pairs or small groups. Careful planning is necessary to ensure that all learners get the opportunity to do the constructions themselves.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
Accurately construct (without a protractor): • bisecting lines and angles • perpendicular lines from a given point • triangles • quadrilaterals By construction investigate: • The properties of triangles	By construction investigate: • The diagonals of rectangles, squares, parallelograms, rhombi and kites • The sum of interior angles of polygons • The minimum conditions for two triangles to be congruent	• Although constructions are not done again in the FET phase, they are a crucial step in the understanding of many of the geometry theorems that are covered right up to Grade 12

GLOSSARY OF TERMS

Term	Explanation / Diagram
Lines	Lines exist within a plane and continue to both sides forever and will have an arrow on both ends. Think of the number lines.
Line segment	A type of line that extents from one point to another point
Ray	A type of line that starts at a certain point and ends with an arrow as it possibly continues forever.
Protractor	A mathematical instrument that is used to draw or measure angles. The mathematical instrument can either be round or half a circle.
Point	A dot representing an exact spot on a plane, can be seen as the starting point of a ray.
Obtuse angle	An angle that is greater than 90° but smaller than 180°
Acute angle	An angle that is smaller than 90°
Reflex angle	An angle that is greater than 180° and smaller than 360°
Straight lines	An angle of exactly 180°
Right angle	An angles of exactly 90°
Revolution	An angle of 360°
Circle	A round shape that does not have any corners and is drawn using a compass
Arc	A curved part or portion of a circle
Segment	The major segment is the bigger part of the circle after a chord has been drawn. The minor segment is the smaller part of a circle after a chord has been drawn
Chord	A line that cuts the circle into two unequal parts
Radius	Half of the diameter. A line that stretches from the centre of the circle to the circumference
Diameter	The line that stretches from one side of the circle to the other going through the centre of the circle that cuts the circle into two halves.
Horizontal Line	A line that is drawn from left to right
Vertical Line	A line that is drawn from the top to the bottom.
Perpendicular Line	This line would meet another line making a 90° angle
Bisect	This means to cut a line or an angle in half
Scalene triangle	A type of triangle that does not have any sides equal to each other and no angles would have the same measured size.
Isosceles triangle	A triangle where two sides are equal and the two base angles to these equal sides are also equal.
Equilateral triangle	A triangle that has all three sides equal and all three interior angles are 60°

Term	Explanation / Diagram
Included angle	The angle that is created by two intersecting rays or lines or line segments.
Right angled triangle	A triangle that has one interior angle that is 90°
Hypotenuse	The side of a right angled triangle that is opposite the 90° angle
Opposite side	The side across from the point or angle being looked at.
Adjacent side	The side right next to the point or angle being looked at
Congruency	This is when figures are exactly the same then they are congruent
Similarity	When shapes are the same shape but are varied in size we can say the shapes are similar
Quadrilaterals	A shape with four sides
Rectangle	A quadrilateral with four right angles and two pairs of parallel sides, the opposite sides are equal in length
Square	A type of rectangle but all four sides are equal in length
Parallelogram	A quadrilateral with two pairs of opposite sides equal and parallel, two opposite angles are acute and two are obtuse.
Rhombus	A type of parallelogram where all four sides are equal in length
Hexagon	A shape with 6 sides
Pentagon	A shape with 5 sides
Congruent	Exactly the same. Identical. Equal sides and equal angles
Similar	Looks the same. Equal angles and sides in proportion

SUMMARY OF KEY CONCEPTS

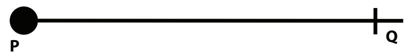
Construction of perpendicular lines

Constructing a perpendicular bisector:

For example: Construct a perpendicular bisector of line PQ with a length of 10cm

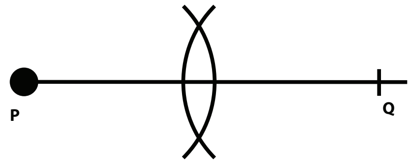
Step 1:

- Draw a line longer than 10cm.
- Use a ruler and compass to draw line PQ on the longer line drawn
- Label the points P and Q



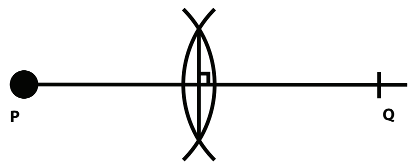
Step 2:

- Place your compass on P and draw an arc more than halfway (more than 5cm in this case) above AND below the line
- Repeat the process from Q, keeping your compass the same length, making sure the arcs cross each other.



Step 3:

- Using your ruler, draw a line joining the two places where the arcs cross.

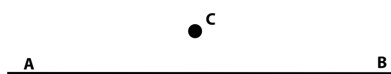


Constructing a perpendicular line from a point above a given line

For example: Construct a line perpendicular to a line AB from point C

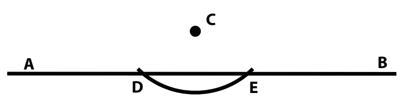
Step 1:

- Draw a line AB and mark a point C anywhere above it



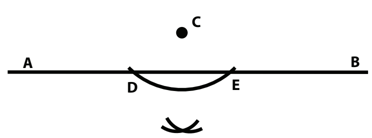
Step 2:

- Place your compass on point C and draw an arc making sure it intersects the line AB in two places. Name these points of intersection D and E



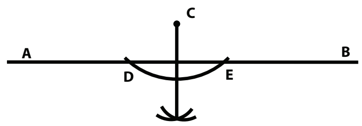
Step 3:

- Place your compass on D and draw an arc below the line AB
- Keeping your compass the same length, repeat the process from E making sure you cross the first arc.



Step 4:

- Draw a line from point C through where the two arcs meet.

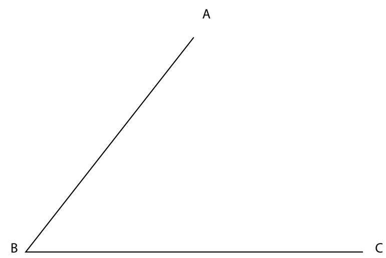


Construction of angles

Bisecting an angle

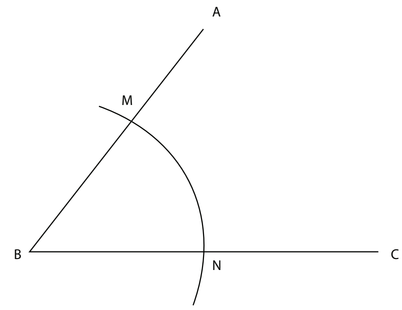
Step 1:

- Draw any size angle ABC



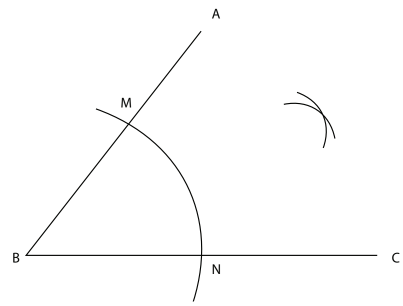
Step 2:

- Place your compass on B and draw an arc crossing both line segments. Call these points M and N



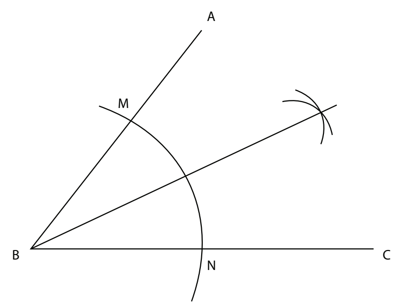
Step 3:

- Place your compass on point M, draw an arc inside the angle
- Keeping your compass the same length, place your compass on N and intersect the arc made from M



Step 4:

- Using your ruler, draw a line from B through the point of intersection of the two arcs.



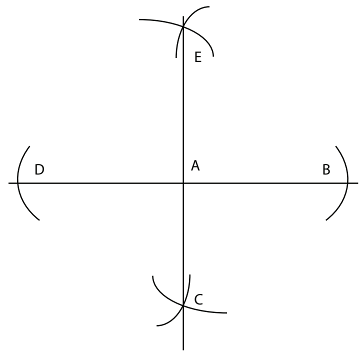
Constructions of a right angle and a 45° angle using a compass

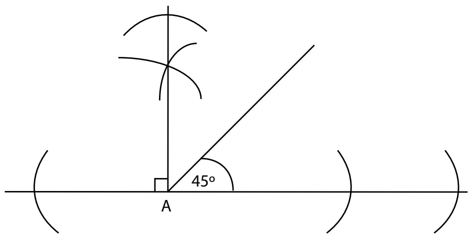
Step 1:

- Follow the instructions on 'Constructing a perpendicular line from a point above a given line'
- This will give a 90^0 angle

Step 2:

- Follow the instructions on 'Bisecting an angle'
- This will give a $45°$





Construction of 60° angles

Step 1:

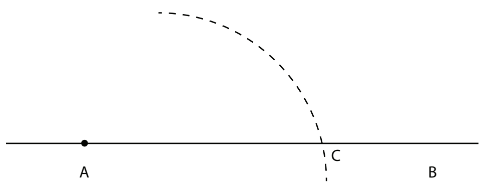
- Draw a line AB of any length
- Place your compass on A, and draw a long arc to intersect AB. Call this point C

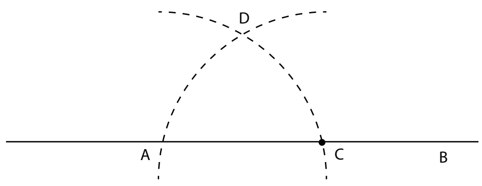
Step 2:

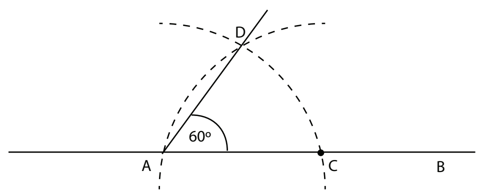
- Keeping your compass the same length, place your compass on C and make an arc across the first arc.
- Call this point D

Step 3:

- Join A to D
- This is a 60° angle







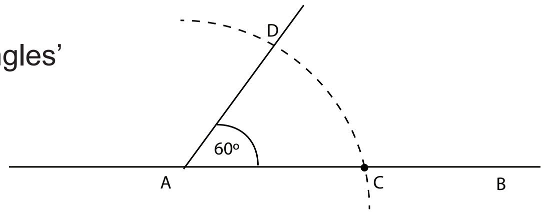
Constructions of a 30° angle using a compass

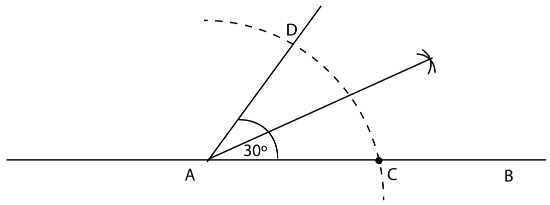
Step 1:

- Follow the instructions ‘Construction of 60° angles’

Step 2:

- Follow the instructions ‘‘Bisecting an angle’
- This will give a 30o angle





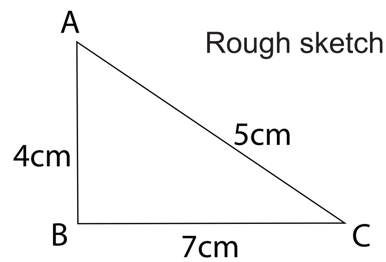
Construction of triangles

ALWAYS draw a rough sketch first, labelling the triangle correctly according to the information given

Construction of a Scalene triangle

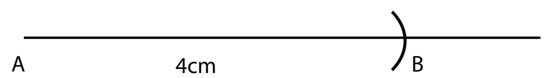
For example: Construct $\triangle ABC$,

with AB = 4cm; AC = 5cm and BC = 7cm



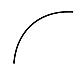
Step 1:

- Draw a line more than 4cm
- measure 4cm accurately using your compass on your ruler
- mark off the exact measurement on the line you drew, marking the points A and B



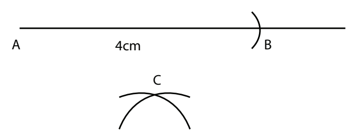
Step 2:

- Set your compass to 5cm, place your compass on B and make an arc.



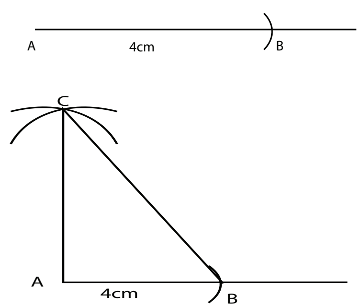
Step 3:

- Set your compass to 7cm, place your compass on A and make an arc. Ensure it crosses the arc from step 2. Call this point C



Step 4:

- Join BC and AC



Construction of an Equilateral triangle

For example: Construct ∆PQR,
with PQ = PR = QR = 6cm

Step 1:

- Follow the steps of 'Construction of a Scalene triangle' but keep your compass measured at 6cm all the time.

Construction of an Isosceles triangle

For example: Construct ∆DEF, with DE = DF = 8cm

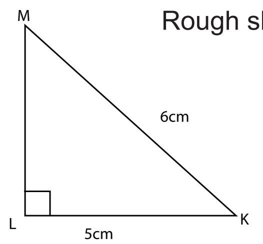
Step 1:

- Follow the steps of 'Construction of a Scalene triangle' using 8cm for the first two measurements then join the final sides.

Construction of a Right-angled triangle

For example: Construct ∆KLM,
with L = 90°, and KL = 5cm and KM = 6cm.

Rough sketch

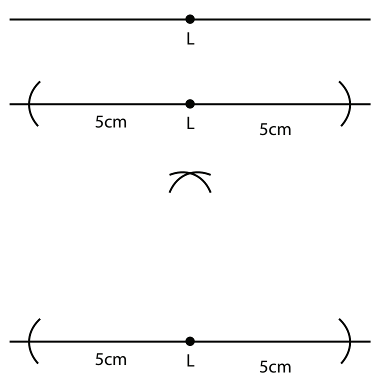


Step 1:

- Draw a horizontal line more than twice the length of the shorter side given.
- Place the point that needs to be the right angle near the centre of the line (in this case 'L')

Step 2:

- Measure the length of the shorter side with the compass and a ruler.
- Place the compass point on the marking for the right angle and mark the length off with an arc on either side (to the left and right) of the point on the horizontal line

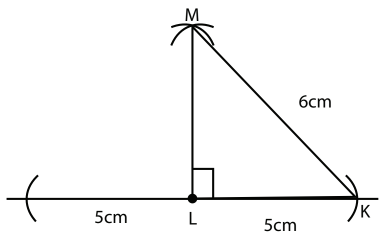


Step 3:

- Measure the length of the hypotenuse using the compass and a ruler.
- Place the compass point on the first marked arc from the previous step and make a new arc in the space above the marked point for the right angle.
- Move the compass point to the other arc made in the previous step and make an arc to cross the first arc just made.

Step 4:

- Use the crossing of the arcs to make the right angle (at 'L' in this case) and also to join up with the other point to form the hypotenuse and hence the triangle



Construction of parallel lines

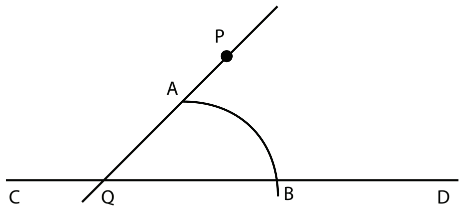
Constructing a line parallel to one already given

Example: Construct a line parallel to CD

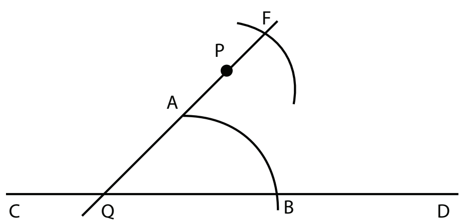
through the point P

Step 1:
Place your compass on Q (the point of intersection of the 2 lines given) and

draw an arc that will intersect both lines. Call these points A and B

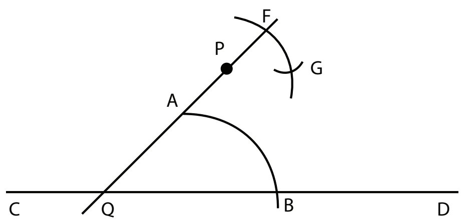


Step 2:
Keeping your compass the same length, place your compass on point P and draw an arc (it will only cross one line). Call this point F

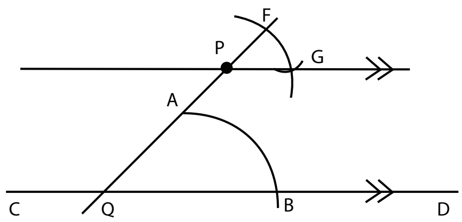


Step 3:
Using your compass, measure out the length of the arc that touches both lines (the one drawn in step 1 - AB) Keeping your compass the same length, place your compass on F and

mark off this length on the arc. Call this point G



Step 4:
Using your ruler, draw a line through PG

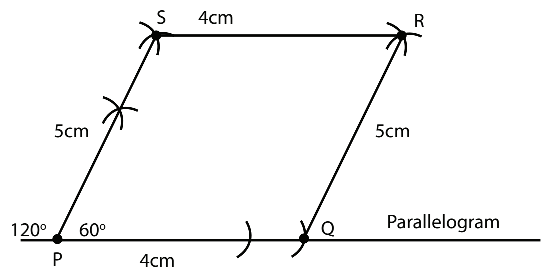


Construction of quadrilaterals

Construction of a Parallelogram

Remember: A parallelogram is a 4 sided figure with opposite sides equal and parallel to each other.

Example: Construct a parallelogram PQRS, with $\hat{P} = 60°$, PQ = 4cm and PS = 5cm



Step 1:

- Draw a rough sketch

Step 2:

- Construct a 60° at P

Step 3:

- Construct a 4cm line from P to Q

Step 4:

- Construct a 5cm line from P to S

Step 5:

- Construct a line through S parallel to PQ

Step 6:

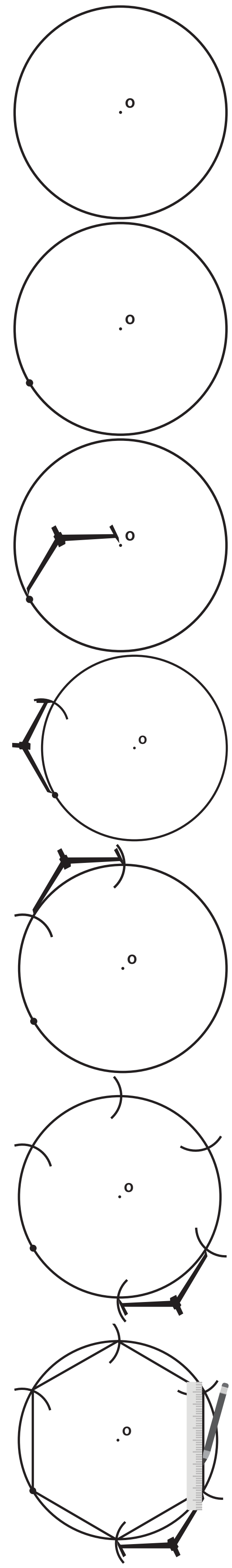
- Construct a line through R parallel to PS

Construction of a Rhombus

Remember: A rhombus is a parallelogram with all 4 sides equal in length so the steps are the same as constructing a parallelogram.

Constructing a hexagon

Follow these instructions to construct a hexagon inside a circle:



Step 1:
- Draw a circle with centre O.

Step 2:
- Mark a point anywhere on the circle.
  This will be the first vertex (corner) of the hexagon

Step 3:
- Set the compasses on this point and set the width of the compasses to the centre of the circle. The compasses are now set to the radius of the circle. Don’t draw anything at this stage – you are just using the compass to measure the distance

Step 4:
- Make an arc across the circle. This will be the next vertex of the hexagon.
- The side length of a hexagon is equal to its circumradius - the distance from the centre to a vertex

Step 5:
- Move the compasses on to the next vertex and draw another arc. This is the third vertex of the hexagon.

Step 6:
- Continue in this way until you have all six vertices.

Step 7
- Draw a line between each successive pairs of vertices, for a total of six lines.

Step 8
- These lines form a regular hexagon inscribed in the given circle.

RESOURCES

The following link has excellent instructions with regards to constructions if access to the internet is possible. Each construction is shown on a board where learners can see what the compass is ‘doing’.

http://www.mathopenref.com/constructions.html

TOPIC 2: GEOMETRY OF 2D SHAPES

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Space and Shape' which counts for 30% in the final exam.
- The unit covers revision of the properties of triangles as well as the theorems from Grade 8 which extends into similar and congruent triangles. Quadrilaterals are also covered again but in more detail.
- It is important to note that unless learners gain a good understanding of all of these aspects of the different shapes studied, coping with geometry in the FET phase will be difficult. One needs to keep in mind that geometry counts one third of the second paper at Grade 12 level.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
Identify and write clear definitions of: • Equilateral triangles • Isosceles triangles • Right-angled triangles Identify and write clear definitions of: • Parallelogram • Rectangle • Rhombus • Square • Trapezium • Kite	• Revision of all previous work concerning triangles and quadrilaterals	• All theorems from the senior phase as well as an understanding of the properties of quadrilaterals are important in the FET phase.

GLOSSARY OF TERMS

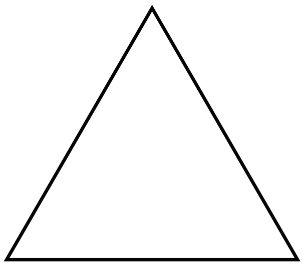
Term	Explanation / Diagram
Triangle	A 3 sided closed shape
Square	A quadrilateral with all angles 90° and all sides equal
Circle	A round plane figure whose boundary (the circumference) consists of points equidistant from a fixed point (the centre).
Quadrilateral	A 4 sided closed shape
Parallelogram	A quadrilateral with two pairs of parallel sides
Rectangle	A quadrilateral with all angles 90° and opposite sides equal and parallel
Rhombus	A quadrilateral with two pairs of parallel sides and all 4 sides equal
Trapezium	A quadrilateral with one pair of parallel sides
Kite	A quadrilateral with two pairs of adjacent sides equal
Equilateral	A triangle with all 3 sides and all 3 angles equal
Isosceles	A triangle with 2 equal sides
Scalene	A triangle with no equal sides
Right-angled triangle	A triangle with one right angle
Acute-angled triangle	A triangle with 3 acute angles
Obtuse-angled triangle	A triangle with one obtuse angle
Parallel lines/sides	Lines exactly the same distance apart at all points. Has the same slope.
Corresponding	In the same position
Congruent	Exactly the same. Identical. Equal sides and equal angles
Similar	Looks the same. Equal angles and sides in proportion

SUMMARY OF KEY CONCEPTS

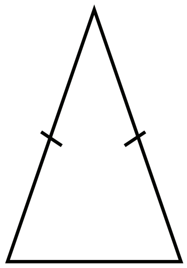
Revision of Grade 8 concepts

1. Classification of triangles

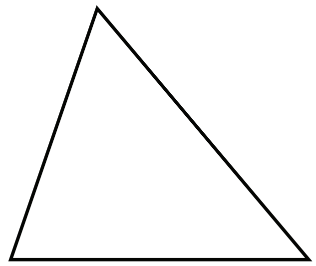
you can classify triangles by their sides



Equilateral Triangle
Has all 3 sides equal

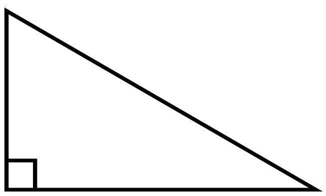


Isosocles Triangle
Has 2 sides equal

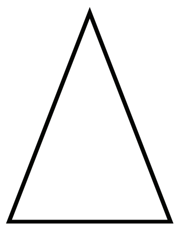


Scalene Triangle
Has no sides equal

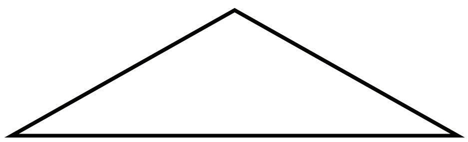
you can also classify triangles by their angles



Right Angle Triangle
Has 1 angle that is a right angle (90^{o})

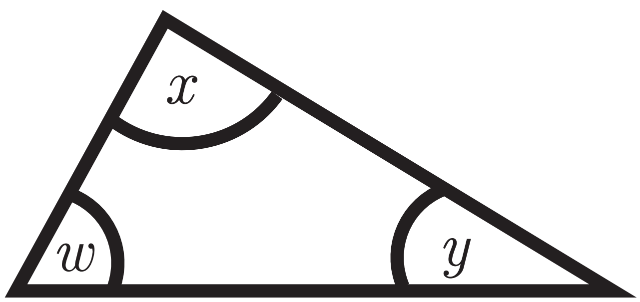


Acute Triangle
Has all 3 angles that are acute angles (less than 90^{o})



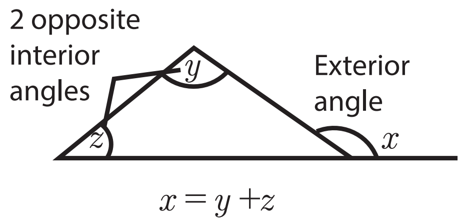
Obtuse Triangle
Has 1 angle that is an obtuse angle (more than 90^{o})

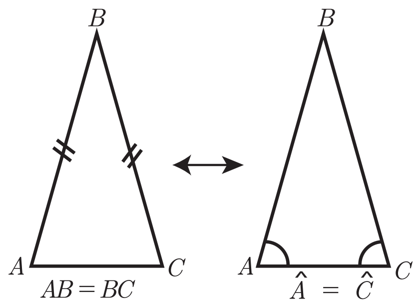
2. Theorems of Triangles



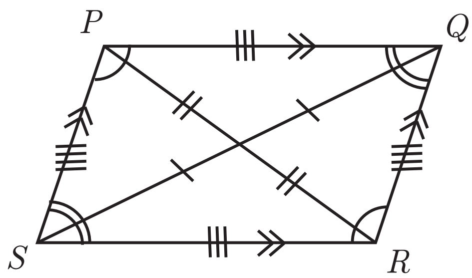
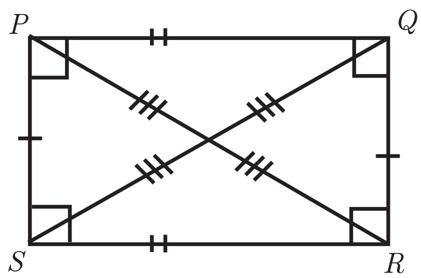
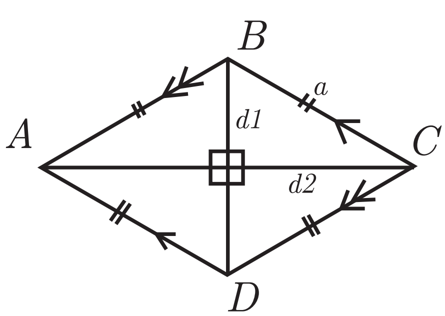
$$w+x+y=180^{o}$$

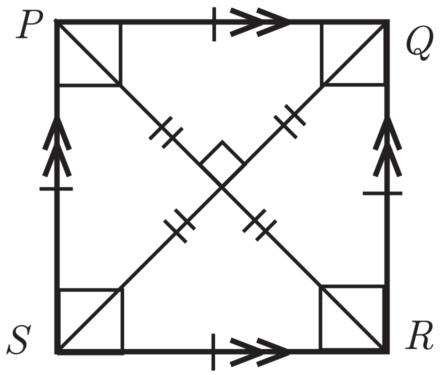
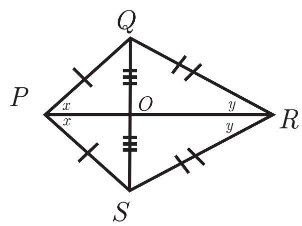
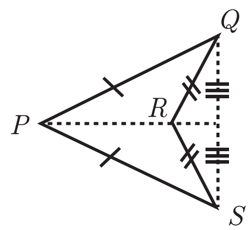
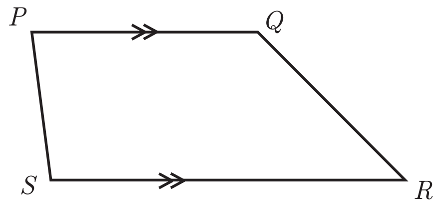
a. The interior angles of a triangle add up to180°
b. The exterior angle of a triangle is equal to the sum of the opposite interior angles
c. In an isosceles triangle the angles subtended from two equal sides are equal





3. Properties of Quadrilaterals

Shape	Example	Characteristics of the quadrilateral
Parallelogram (PARM)		• 2 pairs of opposite sides are parallel. • 2 pairs of opposite sides are equal. • Both pairs of opposite interior angles equal in size. • Diagonals bisect each other when they intersect.
Rectangle (RECT)		• 2 pairs of opposite sides are parallel. • 2 pairs of opposite sides are equal. • Diagonals are equal in length and bisect each other when they intersect. • All interior angles are right angles.
Rhombus (RHOM)		• Both pairs of opposite sides are equal and both pairs of adjacent sides are equal. • Both pairs of opposite sides are parallel. • Opposite angles are equal. • Diagonals bisect each other and bisect perpendicularly. • Diagonals bisect the angles of the rhombus.

Square (SQ)		• Both pairs of opposite and adjacent sides are equal. • All interior angles are $90°$ • Diagonals bisect each other perpendicularly. • Both pairs of opposite sides are parallel
Kite		• Both pairs of adjacent sides are equal. • Diagonals intersect perpendicularly. • One diagonal is bisected by the other. • One diagonal bisects the angles in the kite.
Trapezium (TRAP)		• One pair of opposite sides parallel.

Congruent Triangles

Congruent triangles are exactly the same size. This means that all 6 possible measurements (3 sides and 3 angles) are exactly the same.

There are 4 conditions of congruency, which involve only needing to know that a particular 3 measurements are the same in order to conclude that the triangles are congruent. In other words all 6 measurements are not necessary in order to assume congruency.

To show that two triangles are congruent, the following notation is used:

$$\equiv$$

For example:

$\Delta ABC \equiv \Delta PQR$

(Read: triangle ABC is congruent to triangle PQR)

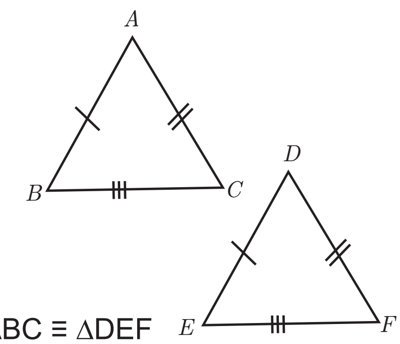
The order is important too. When written in the above format, the following conclusions can be drawn:

- $\hat{A} = \hat{P}$
- $\hat{B} = \hat{Q}$
- $\hat{C} = \hat{R}$
- AB = PQ
- BC = QR
- AC = PR

The 4 conditions of congruency are:

1. SSS – side, side, side

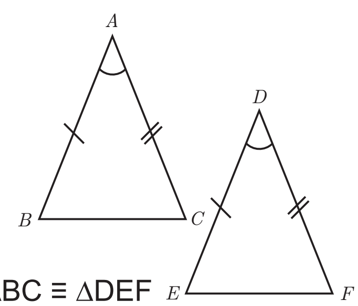
(If 3 sides of one triangle are equal in length to 3 sides of another triangle then the two triangles are congruent)



$\Delta ABC \equiv \Delta DEF$

2. SAS – side, angle, side

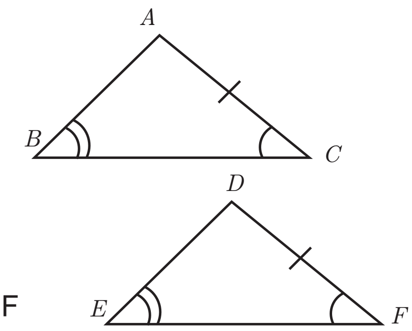
(If two sides and the included angle are equal in length or size to two sides and the **included** angle of another triangle then the two triangles are congruent)



$\Delta ABC \equiv \Delta DEF$

3. AAS – angle, angle, side

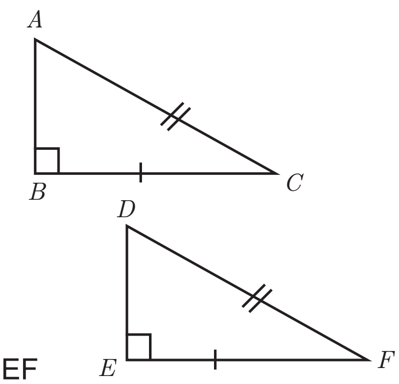
(If two angles and a side in one triangle are equal in length or size to two angles and the **corresponding** side in another triangle then the two triangles are congruent)



$\triangle ABC \equiv \triangle DEF$

4. RHS – right, hypotenuse, side

(If one side and the hypotenuse in a right-angled triangle are the same length as a side and the hypotenuse in another right-angled triangle then the two triangles are congruent)



$\triangle ABC \equiv \triangle DEF$

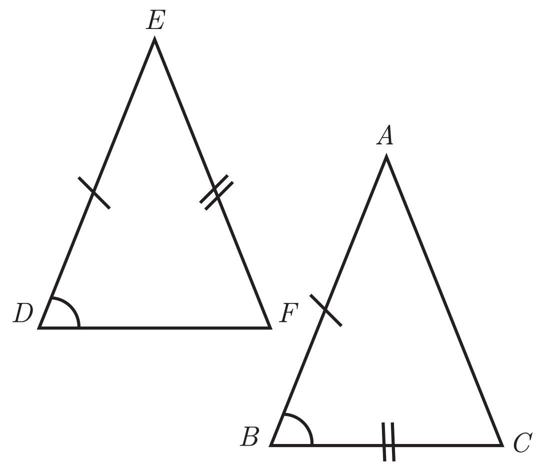
Two of the above conditions of congruency need further explanation as they have extra conditions that need considering before concluding that two triangles may be congruent.

SAS:

If two triangles have two sides of equal length and one angle of the same size they may be congruent.

However, the following needs to be checked as well: Is the angle included? This means, is the angle that is marked equal lying between the two sides marked equal?

The following triangles are **not congruent:**



For these triangles to be congruent, angle E needed to be equal in size to angle B, as these are the two that lie between the two sides marked equal (the angle is included)

Teaching Tip:

Remind learners to always write this condition with the 'A' between the two 'S's' (SAS) to help remind them that the angle must be between the two sides.

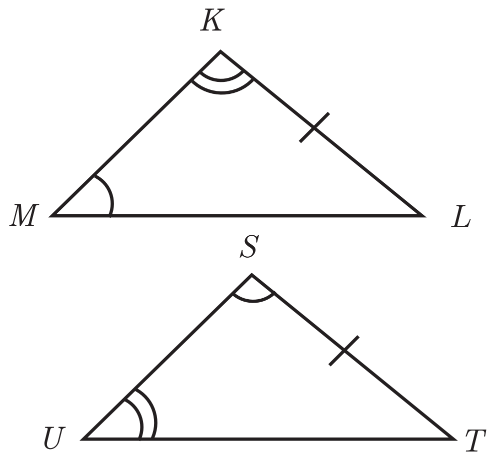
AAS:

If two triangles have two angles equal in size and one side the same length they may be congruent.

However, the following needs to be checked as well: Is the side corresponding in each triangle?

This means, are the sides that are marked equal sitting in the same place in each of the triangles. The side should be opposite the same angle in each of the triangles.

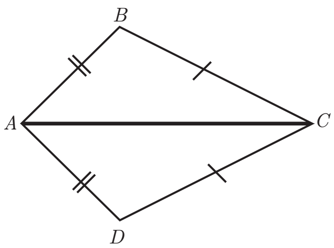
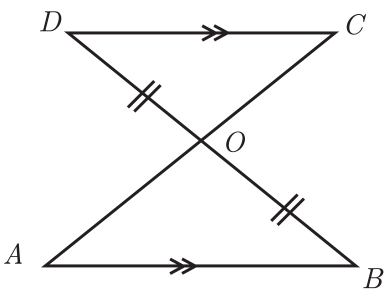
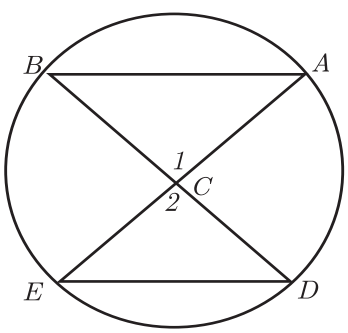
The following triangles are **not congruen**t:



For these triangles to be congruent, TU needed to be equal to KL (they are both opposite the angle marked with a double arc) OR ML needed to be equal to ST (they are both opposite the angle marked with a single arc). Note: KM could have also been marked equal to SU as they are both opposite the angle with no marking.

Teaching Tip:
Point out to learners that within the two conditions they have to be more careful with (SAS and ASA) – it is always the odd one out (A in SAS and S in ASA) that needs to be checked for the extra condition.

Examples of questions:

	Statement	Reason
Prove: $\Delta ABC \equiv \Delta ADC$  	In ΔABC and ΔADC AB = AD BC = CD AC = AC $\Delta ABC \equiv \Delta ADC$	 Given Given Common SSS
 **Prove:** $\Delta ABO \equiv \Delta CDO$	In ΔABO and ΔCDO BO = OD $\hat{A} = \hat{C}$ $B\hat{O}A = C\hat{O}D$ $\Delta ABO \equiv \Delta CDO$ Note: $\hat{B} = \hat{D}$ could have also been used	 Given Alt equal ; AB//CD Vert opp <'s equal AAS
Given Circle with Centre C:  **Prove:** $\Delta ABC \equiv \Delta DEC$	In ΔABC and ΔDEC $\hat{C}_1 = \hat{C}_2$ BC = CD AC = CE $\therefore \Delta ABC \equiv \Delta DEC$	 Vert opp <'s equal Radii Radii SAS

Similar Triangles

Similar triangles have the same shape but are not the same size.

There are two conditions that make triangles similar:

- Sides are in proportion
- All three angles are equal

To show that two triangles are similar, the following notation is used:

///

For example:

$\triangle$ABC///$\triangle$PQR

(Read: triangle ABC is similar to triangle PQR)

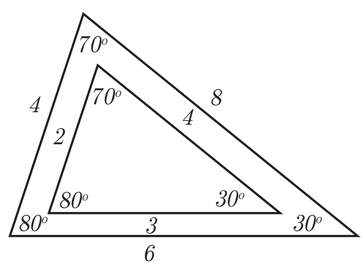
The order is important too. When written in the above format, the following conclusions can be drawn:

- $\hat{A} = \hat{P}$
- $\hat{B} = \hat{Q}$
- $\hat{C} = \hat{R}$
- $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

There are two ways to prove that two triangles are similar to each other:

1. Show that all three angles are equal (In this case it would be enough to find two angles equal as the third one would have to be equal due to the use of the theorem that states all three angles of a triangle add up to 180°)
2. Show that the corresponding sides are in proportion

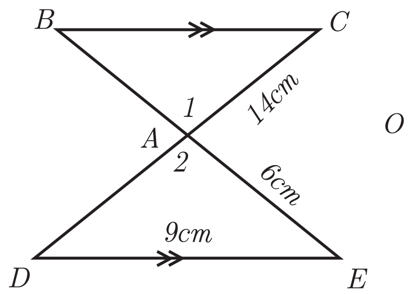
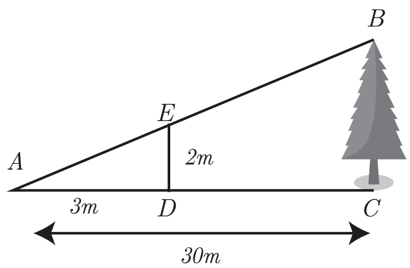
All the corresponding angles are equal to each other.



$\frac{4}{2} = \frac{8}{4} = \frac{6}{3}$ shows that all the corresponding sides are in proportion

Teacher note:
A good understanding of similarity will lead to a good understanding of trigonometry which is covered in the FET phase. Ensure time is spent in developing this understanding.

Example of questions:

1. Prove $\Delta ADE \ \vert\vert\vert\ \Delta ACB$ 2. Find the lengths of AD and AB 	1. In ΔADE and ΔACB $\hat{A}_1 = \hat{A}_2$ $\hat{D} = \hat{C}$ $\therefore \hat{E} = \hat{B}$ $\Delta ADE \ \vert\vert\vert\ \Delta ACB$ $\therefore \frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$ 2. $\frac{AD}{14} = \frac{9}{12} = \frac{6}{AB}$ (Simplify $\frac{9}{12} = \frac{3}{4}$) $\therefore \frac{AD}{14} = \frac{3}{4}$ $\therefore 4AD = 42$ $\therefore AD = 10.5$ $\therefore \frac{3}{4} = \frac{6}{AB}$ $\therefore 3AB = 24$ $\therefore AB = 8$	Vert opp $\angle$'s equal Alt $\angle$'s equal; DE//BC <'s of Δ = 180° (or alt <'s); DE//BC AAA $\Delta ADE \ \vert\vert\vert\ \Delta ACB$
Find the height of the tree (BC) AD = 3m, DE = 2m and AC = 30m (First fill in the points given that are not already on the diagram) 	In ΔABC and ΔAED $\hat{A} = \hat{A}$ $\hat{B} = \hat{E}$ $\hat{C} = \hat{D}$ $\therefore \Delta ABC \ /// \ \Delta AED$ $\frac{AB}{AE} = \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{2} = \frac{30}{3}$ $\therefore 3BC = 30 \times 2$ $\therefore 3BC = 60$ $\therefore BC = 20$ The tree is 20m high	Common Corres <'s ; parallel lines <'s of Δ = 180° AAA

NOTE: When asked to prove congruency or similarity, the triangles will always be written in the correct order (in other words with the equal sides and angles matching up). This should be of great help to the learners as it will give them a good place to start and know which angles and sides they are expecting to be the same size. However, they need to be warned that they cannot just say any of these are equal without giving a geometry reason for saying so.

Summary of most common reasons for sides or angles being equal:

Given	When the information has been given in the question or on the diagram and no actual knowledge is necessary
Common	When a triangle shares a side or angle with another triangle making it the same length or size
Vertically opposite angles equal	This is common when the shape of a bowtie is seen
Radii (plural of radius)	This is common when triangles are drawn inside circles – look out for lines drawn from the centre. Remember that all radii are equal in length in a circle
Alternate or corresponding angles	This is important to look for when any parallel lines are marked anywhere on the diagram

It is essential that learners realise they need to know the theorems they learnt in Grade 8 very well. Without this knowledge, further geometry is almost impossible.

Polygons

All shapes are classified as polygons. The number of sides that a shape has places it into a specific subset of the greater group polygons. Regular polygons have sides that are all the same length and all the interior angles would then also be the same size.

- 3 sided shapes – triangles
- 4 sided shapes – quadrilaterals
- 5 sided shapes – pentagon
- 6 sided shape - hexagon
- 7 sided shape - heptagon
- 8 sided shape – octagon

Formulae that you must know relating to polygons:

- The sum of the interior angles of a polygon $=(n-2)\,180^\circ$ (n is the number of sides the shape has)
- The sum of the exterior angles of all polygons = 360°
- The size of each interior angle of a polygon= $\dfrac{(n-2)\,180^\circ}{n}$

  The size of each exterior angle of a polygon= $\dfrac{360^\circ}{n}$

TOPIC 3: GEOMETRY OF STRAIGHT LINES

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area ‘Space and Shape’ which counts for 30% in the final exam.
- The unit covers perpendicular lines, intersecting lines and parallel lines.
- It is important to note that in Grade 9, learners revise and write clear descriptions of angle relationships on straight lines to solve geometric problems. Learners are expected to give reasons to justify their solutions for every written statement.
- It is important to note that unless learners gain a good understanding of all of these aspects of the different shapes studied, coping with geometry in the FET phase will be difficult. One needs to keep in mind that geometry counts one third of the second paper at Grade 12 level.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
Recognise and describe pairs of angles formed by: • Perpendicular lines • Intersecting lines • Parallel lines cut by a transversal • Solving geometric problems using the relationships between pairs of angles	Recognise and describe pairs of angles formed by: • Perpendicular lines • Intersecting lines • Parallel lines cut by a transversal • Solving geometric problems using the relationships between pairs of angles	• All theorems from the senior phase as well as an understanding of the properties of quadrilaterals are important in the FET phase.

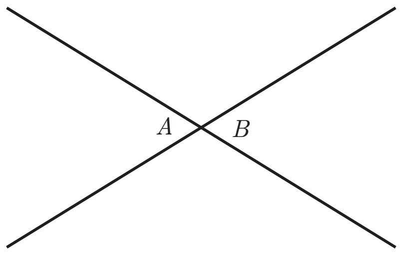
GLOSSARY OF TERMS

Term	Explanation / Diagram
Acute angle	An angle between 0° and 90°
Right angle	A 90° angle
Obtuse angle	An angle between 90° and 180°
Reflex angle	An angle between 180° and 360°
Revolution	A 360° angle
Adjacent	Next to
Complementary	Angles that add up to 90°
Supplementary	Angles that add up to 180°
Vertically opposite angles	Non-adjacent opposite angles formed by intersecting lines
Intersecting lines	Lines that cross each other
Perpendicular lines	Lines that intersect each other at a right angle
Parallel Lines	Lines exactly the same distance apart at all points. Has the same slope.
Corresponding Angles	Angles that sit in the same position
Alternate Angles	Angles that lie on different parallel lines and on opposite sides of the transversal
Co-interior Angles	Angles that lie on different parallel lines and on the same side of the transversal
Transversal	A line that cuts across a set of lines (usually parallel)

SUMMARY OF KEY CONCEPTS

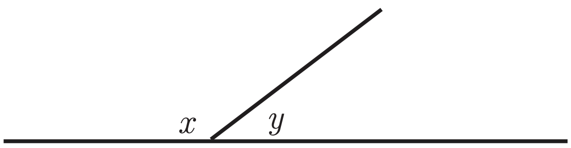
Revision of Grade 8 concepts

1. **Vertically opposite angles are equal**



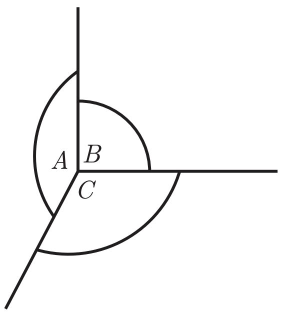
$\hat{A} = \hat{B}$

2. **Adjacent angles on a straight line add up to180o**



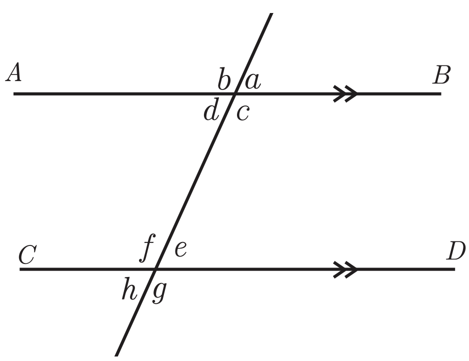
$x + y = 180^o$

3. **Angles around a point add up to360o**



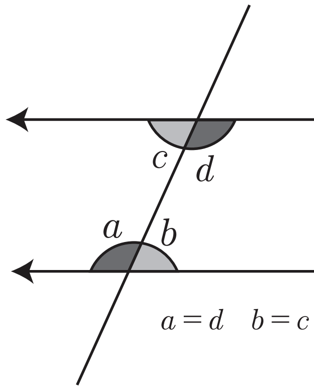
$\hat{A} + \hat{B} + \hat{C} = 360^o$

4. **If parallel lines are cut by a transversal, the corresponding angles are equal**

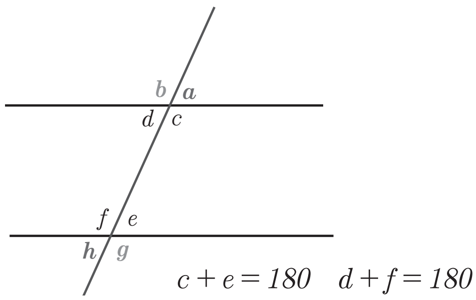


$a = c \quad c = g \quad b = f \quad d = h$

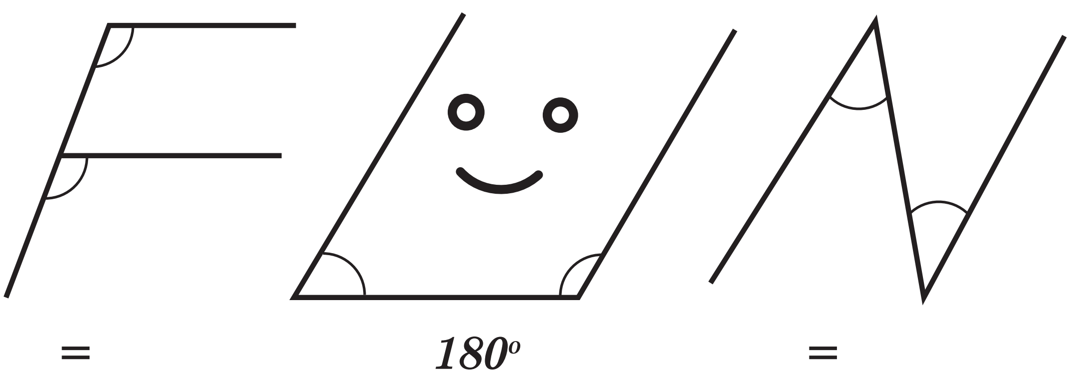
5. **If parallel lines are cut by a transversal, the alternate angles are equal**



6. **If parallel lines are cut by a transversal, the co-interior angles are supplementary**



The following diagram often helps learners to remember the three parallel line theorems



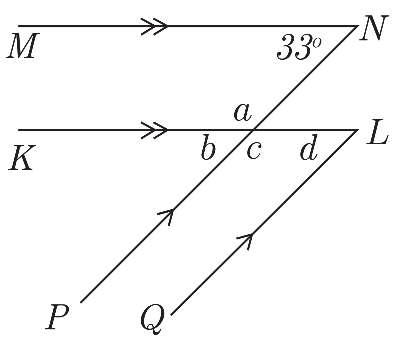
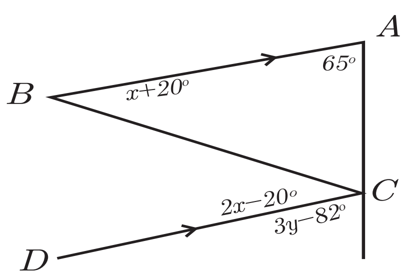
Solving geometric problems

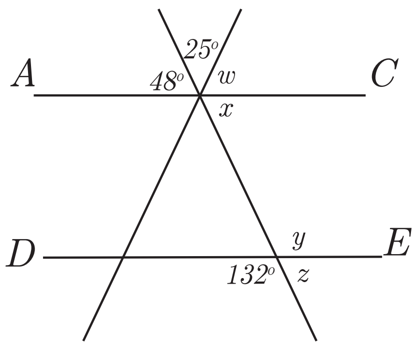
In order to solve general geometric problems it is essential that learners have a good knowledge of all the theorems (rules) they have learnt.

Teaching Tip:

Encourage learners to use colour if possible to mark ‘shapes’ that they see when faced with a diagram full of lines and angles. For example, the ‘Z’ or ‘N’ shape which shows alternate angles. Also, constantly remind learners of the theorems they already know and what they may be looking for in a diagram. Geometry takes practice, but with this and time, all learners are capable of achieving.

Examples:

Find a, b, c and d 	$a + 33° = 180°$ $a = 147°$ $c = 147°$ $b = 33°$ $d = 33°$ Note: Other theorems could have been used in this question For example: Angles on a straight line (a+b or b + c)	MN//KL ; co-int <'s supplementary Vert opp <'s equal MN//KL ; corres <'s equal PN//QL ; corres <'s equal
	$3y - 82° = 65°$ $3y = 65° + 82°$ $37 = 147°$ y $\therefore y = 49°$ $2x - 20° = x + 20°$ $2x - x = 20° + 20°$ $x = 40°$	AB//CD ; corres <'s equal AB//CD ; alt <'s equal

Find w, x, y and z, THEN prove that AC is parallel to DE.  	NOTE: no parallel line theorems may be used to find the unknown angles as we haven't yet proved that the lines are parallel	
	$w + 25° + 48° = 180°$	<'s on str line
	$w + 73° = 180°$	
	$w = 180° - 73°$	
	$w = 107°$	
	$x = 48°$	Vert opp <'s are equal
	$y = 132$	Vert opp <'s are equal
	$z + 132° = 180°$	<'s on str line
	$z = 48°$	
	$\therefore z = x$	
	$\therefore AC \parallel DE$	Corresponding angles are equal

TOPIC 4: THEOREM OF PYTHAGORAS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Measurement' which counts for 10% in the final exam.
- The unit covers revising and applying the Theorem of Pythagoras
- This section should focus on revising this theorem in a way that encourages investigation to lead to a better understanding. All learners should be encouraged to participate.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Determining whether a triangle is right-angled using the theorem • Finding length of missing sides in a right-angled triangle	• Using the theorem to solve problems	• The Theorem of Pythagoras is used extensively in many areas at FET level, including Analytical Geometry, Euclidean Geometry and Trigonometry. Problem solving can also include needing a knowledge of this theorem

GLOSSARY OF TERMS

Term	Explanation / Diagram
Theorem of Pythagoras	In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides
Triangle	A 3 sided closed figure
Hypotenuse	The longest side in a right angled triangle. It is opposite the right angle
Square	To multiply a number by itself. Example: 5 x 5 = 25 (25 is a perfect square)
Square root	Finding what number was multiplied by itself to get a square number Example: $\sqrt{25}$ = 5 (5 is the square root of 25)
Sum of	Addition of
Surd	A root sign Example: $\sqrt{10}$

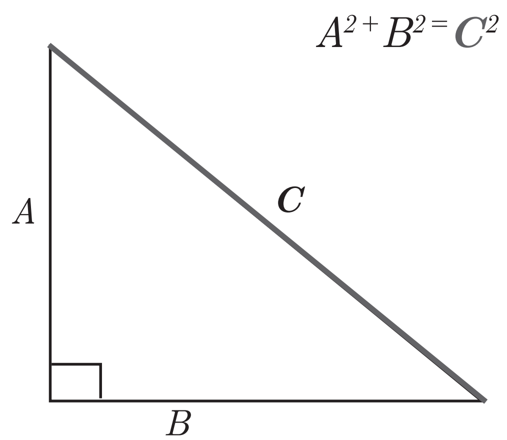
SUMMARY OF KEY CONCEPTS

Revision of Grade 8 concepts

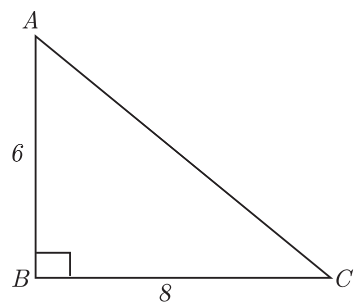
The Theorem of Pythagoras states:

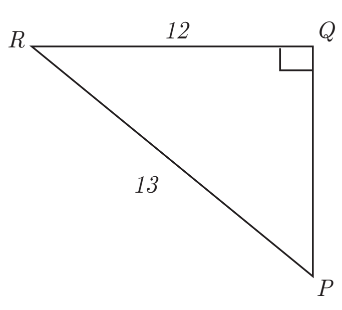
In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides

If two sides are known in a right-angled triangle, it is possible to find the other sides using this theorem.



Basic examples:

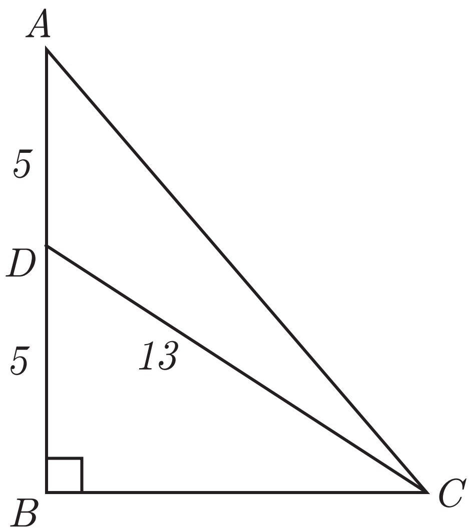




Statement	Reason
$AC^2 = AB^2 + BC^2$ $AC^2 = 6^2 + 8^2$ $AC^2 = 36 + 64$ $AC^2 = 100$ $\sqrt{AC^2} = \sqrt{100}$ $AC = 10$	Pythagoras
$PQ^2 + QR^2 = PR^2$ $PQ^2 + 12^2 = 13^2$ $PQ^2 + 144 = 169$ $PQ^2 = 25$ $\sqrt{PQ^2} = \sqrt{25}$ $PQ = 5$	Pythagoras

Grade 9 examples:

Find AC



Note: It is not possible to find AC unless we know BC so that the theorem of Pythagoras can be used in ΔABC

In ΔCBD — Pythagoras

$$BC^2 + BD^2 = CD^2$$
$$BC^2 + 5^2 = 13^2$$
$$BC^2 + 25 = 169$$
$$BC^2 = 169 - 25$$
$$BC^2 = 144$$
$$BC = 12$$

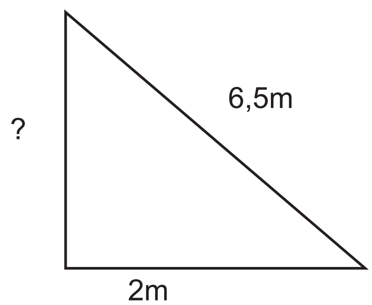
In ΔABC

Pythagoras

$$BC^2 + AB^2 = AC^2$$
$$12^2 + 10^2 = AC^2$$
$$144 + 100 = AC^2$$
$$244 = AC^2$$
$$AC = 15.62$$

A ladder rests up against a wall. The ladder is 6.5m long and its base is 2m from the wall. How high up the wall can the ladder reach?

For questions like this it is always a good idea to make a sketch.

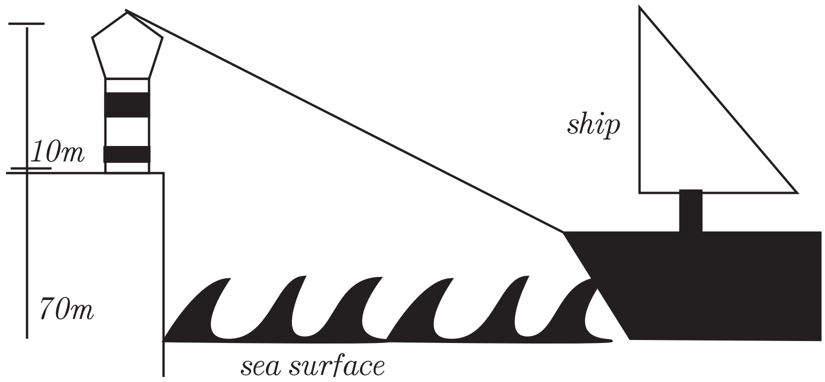


Let the length of the ladder be x

Pythagoras

$$x^2 + 2^2 = (6,5)^2$$
$$x^2 + 4 = 42,25$$
$$x^2 = 42,25 - 4$$
$$x^2 = 38,25$$
$$x = 6,18$$

The ladder can reach 6,18m up the wall



A lighthouse is 10m high and is on top of a cliff that is 70m high. The beam of light from the lighthouse shines a beam of light out to a ship which is 192m from the foot of the cliff. How long is the beam of light?

[First fill in any information that has been given in the question onto the diagram. Also remember that real life questions always have a perfect world in mind. In other words, even though the cliff may not form a perfect right angle to the sea's surface we can assume that it does]

Let the beam of light be called x

$$80^2 + 192^2 = x^2$$
$$6400 + 36864 = x^2$$
$$x^2 = 43264$$
$$\therefore x = \sqrt{43264}$$
$$\therefore x = 208$$

The beam of light is 208m in length.

TOPIC 5: AREA AND PERIMETER OF 2D SHAPES

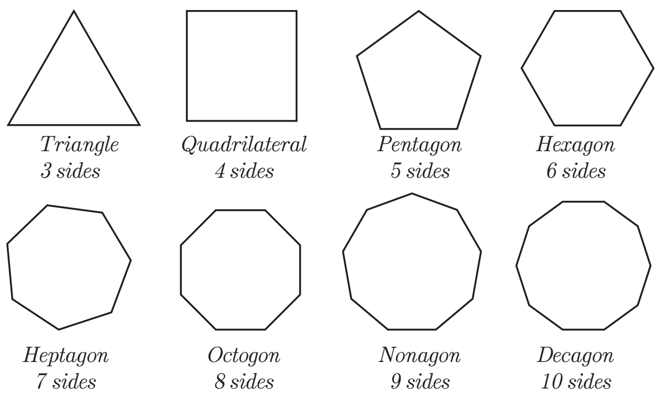
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Measurement' which counts for 10% in the final exam.
- The unit covers the use of appropriate formulae to find the area and perimeter of polygons and circles
- It is important to note that by using formulae correctly in this section learners will use a skill useful in the equations and algebraic expressions sections. Substitution is a skill required in many areas of mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Perimeter and area of squares, rectangles, triangles and circles • Areas of irregular polygons • Use and describe the relationship between the: (i) radius, diameter and circumference of a circle • (ii) radius and area of a circle • Solve problems, involving perimeter and area of polygons and circles • Use and describe the meaning of the irrational number Pi (π) in calculations involving circles • Use and convert between appropriate SI units, including: • $mm^2 \Leftrightarrow cm^2 \Leftrightarrow m^2 \Leftrightarrow km^2$	• Solve problems and calculate perimeter and area of polygons and circles • Investigate how doubling any or all of the dimensions of a 2D figure affects its perimeter and its area	• In the FET phase, measurement becomes part of the geometry and trigonometry sections.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Square	A four sided polygon with all four sides equal in length and all four angles are right angles
Rectangle	A four sided polygon with both pairs of opposite sides equal in length and all four angles are right angles
Triangle	A three sided polygon
Circle	A 2-dimensional shape made by drawing a curve that is always the same distance from a centre.
Polygon	A closed 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. A circle is not a polygon as although it is a closed 2D shape it is not made up of line segments. 
Area	The space taken up by a two dimensional polygon.
Perimeter	The sum of the length of the outside of the shape. The distance around the shape
Diameter	The distance across a circle through its centre
Radius	The distance from the centre of a circle to the circumference
Circumference	The perimeter of a circle. The distance around the side of a circle
Pi (π)	The ratio of the circumference divided by the diameter for ANY circle no matter what its size is.

SUMMARY OF KEY CONCEPTS

Formulae that learners should know and be able to use

Below is a summary table of the all the formulae for perimeter and area that learners are meant to know and also to be able to use.

Note: Learners often struggle with the difference between area and perimeter so it is worth spending time in the beginning ensuring that learners are clear on the difference. This needs to be done before reminding them of the formulae they need to use.

perimeter of a square	P = 4s
perimeter of a rectangle	P = 2(l+b) or P = 2l+2b
area of a square	$A = l^2$
area of a rectangle	A = length x breadth
area of a rhombus	A= length x height
area of a kite	$A = \frac{1}{2} diagonal_1 \times diagonal_2$
area of a parallelogram	A = base x height
area of a trapezium	$A = \frac{1}{2}(sum\ of\ parallel\ sides) \times height$
area of a triangle	$A = \frac{1}{2}(b \times h)$
diameter of a circle	d = 2r
circumference of a circle	c = πd or c=2πr
area of a circle	$A = \pi r^2$

Note: Circumference of a circle is the same as perimeter of a circle

Perimeter

Perimeter is distance and therefore a linear measurement. The unit of measurement used would depend on the size of the shape. For example, a swimming pool's perimeter would be measured in metres (m) but a piece of paper's perimeter would be measured in centimetres (cm) or millimetres (mm).

Useful formulae to calculate perimeter of regular shapes:

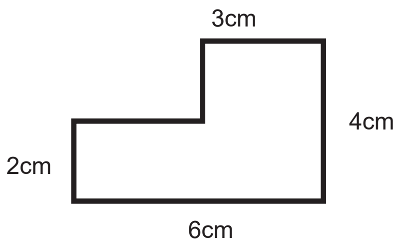
Shape	Formula	Explanation of variables
Square	P = 4s	s = side of the square
Rectangle	P = 2(l+b) or P = 2l+2b	l = length of the rectangle b = breadth of the rectangle
Circle	C = πd or C = 2πr	d = diameter of the circle r = radius of the circle

If the shape is not a regular polygon, it is important that it is looked at carefully to see if all lengths have been covered when adding up to find the perimeter. Also, learners may need reminding of some of the properties of the basic shapes to fill in any sides not marked in the question. For example, opposite sides of a rectangle are equal.

Examples of questions:

a) The top of a can of soup has a diameter of 8cm **What is the circumference of the top of the can?** 	P = πd P = π(8cm) P = 25.13cm NOTE: As the question gave the diameter, the formula for circumference with 'diameter' in it was used. This is not essential but makes it a little easier
b) A bicycle wheel has a radius of 40cm. What distance does the bicycle travel when the wheel makes 10 turns? **Answer in metres.** 	P = 2πr P = 2π(40cm) P = 251.33cm The wheel will travel a distance of ∴ 251.33cm x 10 = 2513.3cm 2513.3 cm = 25.13m

c) Find the perimeter of the following shape:



Note: Learners need to first look at the drawing carefully and fill in the lengths of the missing sides, using information given to help them.

P = 3cm + 4cm + 6cm +2cm + <u>3cm + 2cm</u>
P = 20cm

Area

Area deals with 2-Dimensonal shapes and therefore the measurement is always in 'squared'

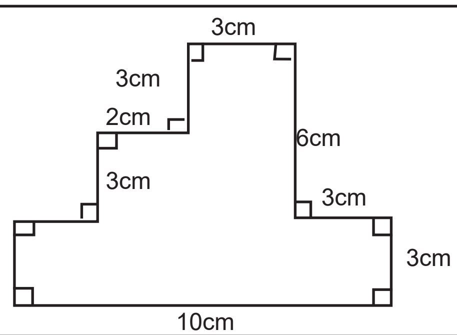
For example: The area of a square with a length of 4cm is 16cm². Whenever 2 dimensions are multiplied the answer must have a square.

Useful formulae to calculate area of regular shapes:

Shape	Formula	Explanation of variables
Square	$A = l^2$	l = side of the square
Rectangle	A = l x b	l = length of the rectangle b = breadth of the rectangle
Triangle	$A = \frac{1}{2}(b \times h)$	b = base of the triangle h = perpendicular height of the triangle
Circle	$A = \pi r^2$	r = radius of the circle

When dealing with more unusual shapes, you need to try and break them up into one or more of the above three in order to find the area or the perimeter of them.

For example:

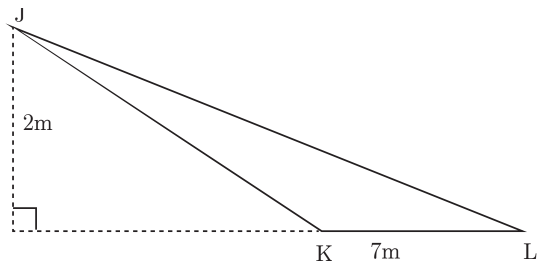


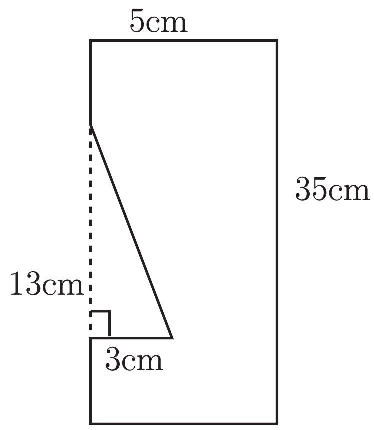
Notice that this shape could be broken into a square at the top and two rectangles underneath that. In other words, to find the area of the polygon:

Area = (3cm)(3cm) + (5cm)(3cm) + (10cm)(3cm)
= $9cm^2 + 15cm^2\ 30cm^2$
= $54cm^2$

Topic 5 Area and Perimeter of 2D Shapes

Examples of questions:

a) Find the area of the following triangle:  NOTE: The height of an obtuse-angled triangle is often measured outside of the triangle itself	area $\Delta JKL = \frac{1}{2} base \times height$ $= \frac{1}{2}(7m) \times (2m)$ $= 7m^2$
b) A clock has a circular face with a diameter of 15cm . **Find the area of the clock's surface.**  NOTE: You have been given diameter but need the radius to find area.	If diameter = 15cm, the radius = 7,5cm Area $= \pi r^2$ $= \pi(7.5)^2$ $= 176{,}71cm^2$
c) The brim of a witch's hat is made by cutting a small circle out of a bigger circle. The bigger circle has a radius of 15cm and the smaller circle has a radius of 7cm. Calculate the area of the brim. 	NOTE: To find the area of the circular piece left over after cutting a smaller circle from a bigger circle, we will first need to find the area of BOTH circles then subtract. Area of the bigger circle $= \pi r^2$ $= \pi(15)^2$ $= 706{,}86cm^2$ Area of the smaller circle $= \pi r^2$ $= \pi(7)^2$ $= 153{,}94cm^2$ Area of the brim $= 706{,}86cm^2 - 153{,}94cm^2$ $= 552{,}72cm^2$

d) Find the radius of a circle with an area of 201cm2	$area = \pi r^2$ fill in what is known $201cm^2 = \pi r^2$ Divide both sides by π $63.98cm^2 = r^2$ square root both sides to get r $\therefore r = 8.31cm$
e) The side of a chair leg is a rectangle with a triangle removed. Find the area of the shape. 	Area of shape = Area of rectangle – Area of triangle $l \times b - \frac{1}{2}bh$ $= (35cm)(5cm) - \frac{1}{2}(3cm)(13cm)$ $= 175cm^2 - 19,5cm^2$ $= 155.5cm^2$

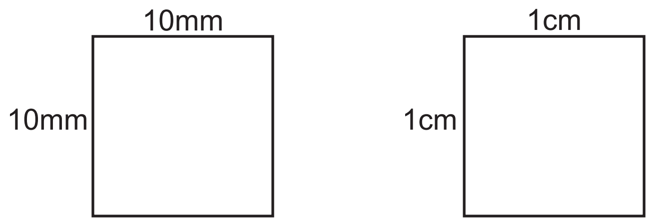
Converting between different units when dealing with area

Consider a linear (length of a line) measurement:

1m = 100cm and 1cm = 10mm and 1km = 1000m

When converting area measurements it isn't as straightforward as multiplying or dividing by 10, 100 or 1000 as it is for linear measurements.

Consider this square:



Area = 10mm x 10mm = $100mm^2$

Area = 1cm x 1cm = $1cm^2$

Since 1cm = 10mm, these squares are the same size, so therefore

$100mm^2 = 1cm^2$

Normally, we would think of the conversion 1cm = 10mm, but since we are dealing with area we need to remember that:

$1cm^2 = 10mm \times 10mm = 100mm^2$

CONVERSION OF AREAS SUMMARY	
$1cm^2 = 100mm^2$	(10 x 10)
$1m^2 = 10000cm^2$	(100 x 100)
$1km^2 = 1000000m^2$	(1000 x 1000)

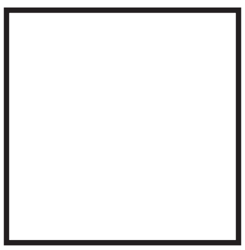
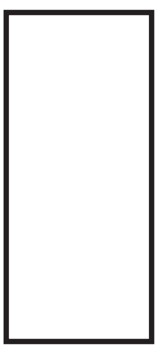
The effect on perimeter or area if dimensions are doubled

This is covered in the investigation which can be found on page 49

INVESTIGATION

The effect of doubling or tripling measurements on area and perimeter

Answer the following questions in the space provided:

SHAPE	PERIMETER	AREA
The following two shapes are the basic ones that all further measurement will depend on. If you are told to change the measurements, it is the measurements of the following two shapes that will be changed.		
SQUARE:  Length = 2cm	Perimeter =	Area =
RECTANGLE:  Length = 3cm Breadth = 2cm	Perimeter =	Area =

Now let's have a look at what happens to the perimeter and area of a shape if the measurements change:		
1. What happens if we double (x2) the length of all the sides?		
Square's new measurements:	Perimeter =	Area =
Rectangle's new measurements:	Perimeter =	Area =
2. What happens if we triple (x3) the length of all the sides?		
Square's new measurements:	Perimeter =	Area =
Rectangle's new measurements:	Perimeter =	Area =
3. What happens if we halve ($\times \frac{1}{2}$) the length of all the sides?		
Square's new measurements:	Perimeter =	Area =
Rectangle's new measurements:	Perimeter =	Area =

Study your answers carefully and answer the following questions:

Consider the perimeter of all shapes.

What happened to the perimeter when the measurements were doubled?

Square:	Rectangle:

What happened to the perimeter when the measurements were tripled?

Square:	Rectangle:

What happened to the perimeter when the measurements were halved?

Square:	Rectangle:

What do you think would happen to the perimeter of a 2D shape if the measurements were multiplied by 10?

Can you write an explanation to generalise what happens to the perimeter when all the measurements are changed?

Consider the area of all shapes.

What happened to the area when the measurements were doubled? (How much bigger is it?)

Square:	Rectangle:

What happened to the area when the measurements were tripled? (How much bigger is it?)

Square:	Rectangle:

What happened to the area when the measurements were halved? (How much bigger is it?)

Square:	Rectangle:

What do you think would happen to the perimeter of a 2D shape if the measurements were multiplied by 10?

Can you write an explanation to generalise what happens to the area when all the measurements are changed?

CONCLUSION

- Effect on perimeter when dimensions are doubled: The perimeter also doubles (x 2) in length.
- Effect on area when dimensions are doubled: The area becomes FOUR (2 x 2) times larger.

- Effect on perimeter when dimensions are tripled: The perimeter also triples (x 3) in length.
- Effect on area when dimensions are tripled: The area becomes NINE (3 x 3) times larger.